Meron excitations in the $\nu = 1$ quantum Hall bilayer and the plasma analogy

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We study meron quasiparticle excitations in the $\nu = 1$ quantum Hall bilayer. Considering the well known single meron state, we introduce its effective form, valid in the longdistance limit. That enables us to propose two (and more) meron states in the same limit. Further, establishing a plasma analogy of the (111) ground state, we find the impurities that play the role of merons and derive meron charge distributions. Using the introduced meron constructions in generalized (mixed) ground states and corresponding plasmas for arbitrary distance between the layers, we calculate the interaction between the construction implied impurities. We also find a correspondence between the impurity interactions and meron interactions. This suggests a possible explanation of the deconfinement of the merons recently observed in the experiments.

] The $\nu = 1$ quantum Hall bilayer [1] is the subject of intensive experimental and theoretical investigations [2]. In 1995, the pseudospin theory of the bilayer [3] was advanced for this system. The theory introduced meron - a new type of quantum Hall quasiparticle. Nevertheless, even today it is not known how a construction of a pair of merons looks [4]. Understanding of that would bring us closer to the understanding of increased susceptibility to the presence of disorder of the neutral superfluid in the pseudospin channel of the bilayer. Namely because of the persistent dissipation in the counterflow measurments [5, 6], there is a wide-spread belief that even in the presence of a moderate amount of disorder, merons – vortices of the superfluid are liberated, dissociated from one another [2, 7, 8]. On the other hand, since Laughlin's seminal paper [9], the plasma analogy has proven to be a very useful concept in analyzing quasiparticle state properties.

In this paper we develop a description of the meron excitations of the pseudospin theory in the longdistance limit. We use the plasma techniques of [10], and argue that meron presence introduces a new type of impurity in the plasma analogy of the so-called (111) ground state. That enables us to easily derive meron charge distributions in the longdistance limit and infer how the construction of a pair of merons would look like in the same limit.

We also consider the same constructions in mixed, composite boson - composite fermion, ground states [11], proposed as a way to capture in the ground state description the effect of quantum fluctuations at finite d, distance between the layers. Charge screening of the single meron construction in the plasma analogy of mixed states is almost without change with respect to the (111) case. But the strength of the $\ln(r)$ interaction in the plasma analogy between a pair of merons gets reduced, being propotional to the density of bosons that decreases as a function of d. Because of, as detailed below, a formal correspondence between the interaction laws between merons and the interaction laws between impurities, this is very suggestive of a mechanism, which (with composite fermion screening; see below) might be responsible for a confinement weakening. Together with disorder the mechanism may lead to the deconfinement believed to exist in the experiments [2].

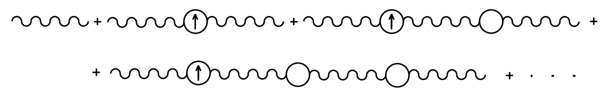

FIG. 1: Diagrammatic summation leading to the $\uparrow$ charge distribution away from an impurity

In the following we will introduce plasma techniques [10] for the (111) state. Because of the unusual nature of the statistical model based on the (111) state implied by the Laughlin prescription [9], it is not clear whether they are valid in this case, but we will show that indeed they can capture the leading longdistance behavior. Let us begin with the most obvious generalization of the Laughlin quasihole construction for the case of the two-component, $\uparrow$ and $\downarrow$, (111) state,

$$\Psi(w) = \prod_{i=1}(w - z_{\uparrow,i})\Psi_{111}(z_\uparrow, z_\downarrow), \tag{1}$$

where the (111) state is,

$$\Psi_{111}(z_\uparrow, z_\downarrow) = \prod_{i<j}(z_{i,\uparrow} - z_{j,\uparrow}) \prod_{k<l}(z_{k,\downarrow} - z_{l,\downarrow}) \prod_{p,q}(z_{p,\uparrow} - z_{q,\downarrow}) \tag{2}$$

(with omitted Gaussian factors). To get the charge distributions at point r away from the center w of the excitation, we use an effective plasma expansion summing only contributions that can be symbolically represented as the ones depicted in Fig. 1. In the Figure, in the Fourier space, the wriggly line is the 2D Coulomb plasma interaction, $-\frac{2\pi}{|\vec{q}|^2}$, and the vertices are: the empty circle of the value of the total density n - the static structure factor of a Bose gas, and the circle with up arrow denotes the density $n_\uparrow$ of the up particles. We probe the $\uparrow$ charge density at r otherwise, if we probe $\downarrow$ density the corresponding sum does not have the first contribution in Fig. 1. Therefore, to get the $\uparrow$ and $\downarrow$ charge distributions, we have to Fourier transform ($\int d^2\vec{q} e^{i\vec{q}(\vec{w}-\vec{r})}[\cdots]$) the following expressions in which $V(\vec{q}) = -\frac{2\pi}{|\vec{q}|^2}$,

$$\rho_\uparrow(q) = V(q) + \frac{V(q) n_\uparrow V(q)}{1 - nV(q)}, \tag{3}$$

and

$$\rho_\downarrow(q) = \frac{V(q)n_\uparrow V(q)}{1 - nV(q)}, \tag{4}$$

for $\uparrow$ and $\downarrow$ charge respectively. We immediately see that the total charge is screened,

$$\rho_c(q) \sim \rho_\uparrow + \rho_\downarrow = \frac{V(q)}{1 - nV(q)}, \tag{5}$$

and $\lim_{q\to 0} \rho_c(q) = Const$, like in the usual Laughlin quasihole case, but the pseudospin charge $\rho_s(q) \sim \rho_\uparrow - \rho_\downarrow = V(q)$ is unscreened growing as $\ln(r)$ (if $w = 0$) with distance r. Therefore the capacitive energy defined as

$$E_c = \int d^2\vec{r}(\rho_\uparrow - \rho_\downarrow)^2, \tag{6}$$

which is in the first approximation proportional to the energy to excite the quasihole, is proportional to (up to logarithmic factors) the area of the system. This is the conclusion of the numerical study in [4]. Therefore the plasma analogy is able to reproduce the main result of the detailed investigation [4] (which helps us to eliminate from further consideration the constructions of the form in Eq.(1) as relevant excitations for the bilayer). The agreement does not come as a surprise if we analyze more closely the diagrams in Fig. 1. In them we are justifiably using the screening properties of the charge channel, which behaves as a plasma. Because of this, from now on, we will refer to the statistical model based on the (111) state as plasma.

In the second quantization formalism the meron excitation of the pseudospin theory [3] that parallels the construction in Eq.(1) is

$$|\Psi_m(w = 0)>= \prod_{m=0}^{N-1} (c^\dagger_{m+1,\uparrow} + c^\dagger_{m,\downarrow})|0> . \tag{7}$$

$c^\dagger_{m,\sigma}$'s create the lowest Landau level (LLL) states, $\Phi_m = \frac{z^m}{\sqrt{2\pi 2^m m!}} \exp\{-\frac{1}{4}|z|^2\}, m = 0, \ldots, N-1$. ($N$ is the number of particles in the system.). In the first quantization description of Eq.(7), $\uparrow$ orbitals are shifted (in the expansion of the Slater determinant of a filled LLL) in the following manner:

$$\Phi_m(z_\uparrow) \to \frac{z_\uparrow}{\sqrt{2(m+1)}}\Phi_m(z_\uparrow). \tag{8}$$

This is a nontrivial change and can not be described by a simple multiplication operation on the ground state like in Eq.(1). When $|z_\uparrow| \to \infty$, more precisely when $m = N-1 \to \infty$ (i.e. m is the last orbital in the ground state) $|\Phi_m(z_\uparrow)|^2$ behaves, in the first approximation, like a delta function, at $|z_\uparrow| = \sqrt{2(m+1)}$, and, in this sense, we can approximately take for multiplying $z_\uparrow$ in Eq.(8), $z_\uparrow = \exp\{i\phi\}\sqrt{2(m+1)}$. Then the excitation, very far from the origin, looks like

$$\prod_{i=1}^{N} \begin{bmatrix} \exp\{i\phi_i\} \\ 1 \end{bmatrix} \Psi_{111}(z_\uparrow, z_\downarrow) \tag{9}$$

in the first quantization [3]. To get a change in the charge distribution away from the origin we need further corrections to the limit in Eq.(9). We assume that they can be described by an expansion in the powers of $1/(|z_\uparrow|)$, and we seek the coefficient of the first correction by solving the following equation with $|z| \equiv r$:

$$\int_{\sqrt{2(m+1)}}^{\infty} dr\ r\ |z|^2|\Phi_m(z)|^2 = 2(m+1)\int_{\sqrt{2(m+1)}}^{\infty} dr\ r\ |\Phi_m(z)|^2 + 2(m+1)\ C\ \int_{\sqrt{2(m+1)}}^{\infty} dr \frac{r|\Phi_m(z)|^2}{r}. \tag{10}$$

The condition and implied expansion are appropriate when we look for charge density distributions of the state in Eq.(7). In the limit $m \to \infty$ we get $C = 0.8$ as can be seen in Fig. 2. In Eq.(10) the searched for correction is for the orbital Φ_m right at the longdistance cut-off $R = \sqrt{2(m+1)} = \sqrt{2N}$ (i.e., the radius of the system). Therefore, in this approximation, in the approach with fixed up and down number of particles, the $\uparrow$ charge density of the excitation in Eq.(7) can be extracted from the following integral, with $|z_{\uparrow 1}| \equiv r$,

$$\rho_{w=0}(r) \sim \int d^2 z_{\uparrow 2} \cdots \int d^2 z_{\downarrow N} \exp\{\sum_i \frac{C}{|z_{i\uparrow}|}\}|\Psi_{111}(z_\uparrow, z_\downarrow)|^2, \tag{11}$$

and analogously for the $\downarrow$ charge density. In this way, in the (111) plasma, we consider a new type of impurity which connects via the interaction $C/(|z_{i\uparrow}|)$ to the $\uparrow$ particles of the plasma. *In this sense* we can propose the following longdistance form of the meron excitation

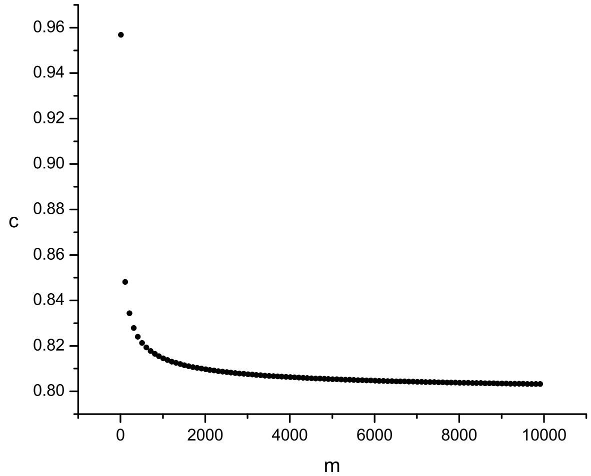

FIG. 2: Dependence of C (Eq.(10)) on $m = N-1$ (the first value at $m = 10$)

at some point $w \neq 0$ in general,

$$\prod_i \frac{z_{i\uparrow} - w}{|z_{i\uparrow} - w|} \exp\{\sum \frac{C}{2|z_{i\uparrow} - w|}\} \cdot \Psi_{111}(z_\uparrow, z_\downarrow). \quad (12)$$

This construction can be easily generalized to the case when there are more than one meron (of both vorticities).

To get the charge distributions ($\uparrow$ and $\downarrow$) far away from the center of the excitation, we use the same type of the approximation introduced in the beginning (Fig. 1) with only one difference. Namely, we change the way impurity connects to the plasma by switching from $V(q) \sim \frac{1}{q^2}$ to $V_m(q) \sim \frac{1}{q}$. In this way $\rho_\uparrow(q) \approx \frac{1}{2}V_m(q)$ and $\rho_\downarrow(q) \approx -\frac{1}{2}V_m(q)$ in the $q \to 0$ limit and for $n_\uparrow = n_\downarrow$, resulting in $E_c \sim \ln R$, where R is the radius of the system, for the energy to excite a meron, in agreement with the XY model considerations and pseudospin theory [3].

By considering the new impurities in the (111) plasma and applying the plasma techniques, we can prove the usual XY model logarithmic interactions between them, which is a result without an obvious connection with the physics and XY model of the bilayer. By considering also a pair of the old impurities (that follow from the construction in Eq.(1)), with same charge and opposite vorticity, we can find that their interaction energy in the plasma grows quadratically as a function of distance. It was found in [4], in numerics, that their real (capacitive) interaction energy behaves in the same way. This all again shows that whenever we have an underlying bosonic analogy and corresponding quasiparticles, like for the Laughlin states [12], or transparent, like in the bilayer case [13], the corresponding plasmas have impurities with identical interaction energy laws, up to the value of couplings, to the interactions among quasiparticles in the quantum Hall systems.

The mixed states proposed as the ground states [11] at finite (not small) d as mixtures of composite bosons of the (111) state and composite fermions of the nearby phase of two decoupled Fermi-liquid-like states can be expressed as

$$\begin{aligned}\Psi_o = \mathcal{P}\mathcal{A}\{&\textstyle\prod_{i<j}(z_{i\uparrow} - z_{j\uparrow})\prod_{k<l}(z_{k\downarrow} - z_{l\downarrow})\prod_{p,q}(z_{p\uparrow} - z_{q\downarrow})\\ &\Phi_f^\uparrow(w_\uparrow, \overline{w}_\uparrow)\textstyle\prod_{i<j}(w_{i\uparrow} - w_{j\uparrow})^2\Phi_f^\downarrow(w_\downarrow, \overline{w}_\downarrow)\prod_{k<l}(w_{k\downarrow} - w_{l\downarrow})^2\\ &\textstyle\prod_{i,j}(z_{i\uparrow} - w_{j\uparrow})\prod_{k,l}(z_{k\uparrow} - w_{l\downarrow})\\ &\textstyle\prod_{p,q}(z_{i\downarrow} - w_{q\uparrow})\prod_{m,n}(z_{m\downarrow} - w_{n\downarrow})\ \}.\end{aligned} \quad (13)$$

z's and w's denote bosons and fermions respectively, $\Phi_f^\sigma, \sigma = \uparrow, \downarrow$ are two filled-Fermi-sea wave functions, $\mathcal{P}$ is the projection to LLL, and $\mathcal{A}$ is the antisymmetrizer for bosons and fermions in each layer separately. The portion of composite fermions increases as d increases. Extracting the number of flux quanta - the number of particles relations from Eq.(13), we can find that the number of up and down composite fermions must be the same. The mixed states are very close to the exact diagonalization ground states. In the following we will apply on them the weakly-screening plasma approach [10], which, again, in the longdistance approximation, was able to reproduce the basic physics of the Fermi-liquid-like composite fermion states. Because of the presence of Φ_f's, any vertex representing connection through composite fermions is effectively the static structure factor of free Fermi gas i.e. $s_\sigma(q) \sim q$ in the small momentum limit.

We can consider excitations of the type described by Eq.(12) in a mixed state. Let us consider the following dipole construction of the introduced excitation:

$$\prod_i \frac{z_{i\uparrow}}{|z_{i\uparrow}|}\prod_j \frac{w_{j\uparrow}}{|w_{j\uparrow}|}\prod_k \frac{\bar{z}_{k\downarrow}}{|z_{k\downarrow}|}\prod_l \frac{\bar{w}_{l\downarrow}}{|w_{l\downarrow}|}\exp\{\frac{C}{2}(\sum_i \frac{1}{|z_{i\uparrow}|} + \sum_j \frac{1}{|w_{j\uparrow}|} - \sum_k \frac{1}{|z_{k\downarrow}|} - \sum_l \frac{1}{|w_{l\downarrow}|})\}\ \Psi_o. \quad (14)$$

We will take that the number of $\uparrow$ and $\downarrow$ bosons is the same and neglect the antisymmetrizer in Ψ_o . Then, the charge distribution of $\uparrow$ charge can be found, first considering bosonic part, which has only a single connection and contribution $\sim V_m(q)$ because of the alternating sign in Eq.(14) and fermionic part that we symbolically depicted in Fig. 3. In Fig. 3 the doubly wriggly line denotes $V_m(q)$, and the crossed circle, twice the static

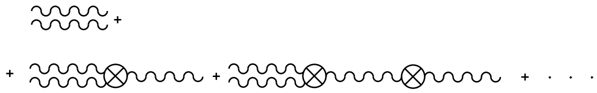

FIG. 3: Diagrammatic summation leading to the fermionic part of the charge distribution away from the dipole (Eq.(14))

structure factor, $s_\uparrow(q)$, of the Fermi gas. Though simple, the final contributions are results of massive cancellations that follow from different interaction signs in the way quasiparticles connect to the plasma. Examining the contributions, we find that despite the presence of the composite fermions and their screening, the $\uparrow$ ($\downarrow$) charge distributions (effectively bosonic) are again propotional to $+(-)\frac{1}{r}$, leading to the infinite energy requirement for the excitation in the thermodynamic limit. The presence of the composite fermions signifies the quantum fluctuations as the distance between the layers increases. Nevertheless, we can again conclude, the excitations in the pseudospin channel (no charge, only vorticity), like the one we considered, retain their meron confinement property [3].

This conclusion is corraborated by a calculation of the plasma interaction among the meron pair of opposite vorticity but the same charge in a mixed state. We state the final result,

$$V_{int}(q) = \frac{V_m^2(q)V(q)(n_\uparrow n_\downarrow + n_\uparrow s_\uparrow(q) + n_\downarrow s_\downarrow(q))}{1 - V(q)(n + 2s(q))} + \frac{V_m^2(q)s^2(q)}{1 - V^2(q)(2s(q))^2}\,\{nV^2(q) + \frac{n^2 + n(2s_\uparrow(q) + 2s_\downarrow(q))}{1 - V^2(q)(2s(q))^2}\,V^3(q)\}, \tag{15}$$

where $s_\uparrow(q) = s_\downarrow(q) = s(q)$ and $n_\uparrow, n_\downarrow$, and $n = n_\uparrow + n_\downarrow$ denote *bosonic* densities. This can be obtained straightforwardly with the help of diagrams. In the $q \to 0$ limit we have,

$$V_{int}(q) \to (-)V_m^2 \frac{n_\uparrow n_\downarrow}{n} + (2\frac{n_\uparrow n_\downarrow}{n^2} - \frac{1}{2})V_m^2(q)s(q), \tag{16}$$

i.e. the leading is the attractive $\ln(r)$ interaction, and the correction is a $\frac{1}{r}$ interaction due to the screening by composite fermions that vanishes in the $n_\uparrow = n_\downarrow$ case [14]. This is a result in the formal setting of plasma analogy, but very likely, due to the mentioned correspondence, also a relevant conclusion for the interaction between two merons in the quantum Hall system. Then please note again that $n_\uparrow, n_\downarrow$, and n, are not overall densities but reduced, due to the presence of fermions, bosonic densities. Therefore, though the type of interaction ($\ln(r)$) stays the same, the coupling strength is weaker due to its proportionality to the density of bosons.

Certainly, it is appropriate to check the amounts of the screening charges of a single meron construction in a mixed state that is the generalization of the construction in Eq.(12). Again with the help of diagrams, they can be easily found, and we will just state their limiting, $q \to 0$, behavior:

$$\rho_\uparrow(q) \to \frac{V_m(q)}{2} + \frac{n_\uparrow - n_\downarrow}{n} V_m(q), \tag{17}$$

and,

$$\rho_\downarrow(q) \to \frac{V_m(q)}{2} + \frac{-2n_\downarrow}{n} V_m(q). \tag{18}$$

Therefore, in the $n_\uparrow = n_\downarrow$ case, the limits do not differ from the case without composite fermions.

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